Boltzmann Machines

A Boltzmann Machine is a probabilistic graphical model defined on a complete graph of binary variables. The graph is partitioned into “visible” units $v$, where values are observed during training and “hidden” units $h$, where values must be inferred. The probability of observing a state in the Boltzmann Machine is governed by its energy function

$$E(v, h) = -\sum_i W_i v_i h_i - \sum_{ij} B_{ij} v_i h_j - \sum_i L_i v_i$$

We study models with different graphical structures between the hidden units.

- Chain structured hidden connections, which are easy to sample from.
- Chimera structured hidden connections, which reflect the architecture of the D-Wave machine.

Units intended to be realized on the hardware are marked with a small diagonal bar.

Quantum Annealing

Quantum Annealing solves problems by encoding them in a physical system

$$F[\rho] = \text{tr} (\rho V) + \lambda \text{tr} (\rho J) + \frac{1}{T} \text{tr} (\rho \ln \rho)$$

- $\rho = |x \rangle \langle x|$ is the density matrix, where $x$ is a distribution over the state of the system. If the state space has $n$ bits then $\rho$ has dimensions $2^n \times 2^n$.
- $V$ is the problem Hamiltonian, which subsumes $W$, $U$ and $L$ in a Boltzmann Machine.
- $K$ is a quantum disordering term which is minimized when all states are in uniform superposition.
- $\lambda$ is a scalar annealing parameter.
- $T$ is the temperature.

In the D-Wave machine...

- $V$ is set by the user.
- $T$ is large at the beginning of computation and small at the end.
- The classical term is very small ($T \approx 0$).

D-Wave Hardware

The D-Wave hardware realizes an Ising model with a type of graphical structure called a Chimera. A Chimera $(M, N, L)$ graph is formed by connecting an $M \times N$ grid of $L \times L$ dense bipartite graphs. The picture on the left shows the connectivity pattern of a Chimera $(3, 3, 4)$ graph.

The D-Wave machine realizes a larger Chimera $(4, 4, 4)$ graph. Future versions of the hardware will implement larger Chimera graphs.

Selected References